② ELIMINATION

$$4x+3y=1$$
$$4(-x+2y)=(8)4$$

$$4x+3y=1$$
$$-4x+8y=32$$
$$\overline{\qquad 11y=33 \qquad}$$
$$y=3$$

$$\downarrow$$

	x	y =	c	
①	4	3	1	
②	-1	2	8	$\leftarrow R_2 * 4$
①	4	3	1	
②	-4	8	32	
②	0	11	33	$\leftarrow R_2+R_1$
	0	1	3	$\leftarrow \frac{1}{11}R_2$

$$0x+1y=3$$
$$y=3$$

$$2(4x+3y)=2(1)$$
$$-3(-x+2y)=-3(8)$$

$$8x+6y=2$$
$$3x-6y=-24$$
$$\overline{\qquad 11x=-22 \qquad}$$
$$x=-2$$

A MATRIX IS IN ROW ECHELON FORM (REF) IF

1. THE LEFTMOST NON-ZERO ENTRY IN EACH ROW
   ↳ 'LEADING' ENTRY
   IS 1.

2. THE LEADING ENTRY IN EACH ROW (EXCEPT ROW 1) IS TO THE RIGHT OF THE LEADING ENTRY OF THE ROW ABOVE IT.

3. ANY ROW WITH ALL 0 ENTRIES IS BELOW ALL ROWS WITH NON-0 ENTRIES

A MATRIX IS IN REDUCED ROW ECHELON FORM (RREF) IF

1. IT IS IN ROW ECHELON FORM

2. ANY COLUMN THAT CONTAINS A LEADING 1 CONTAINS 0 IN ALL OTHER ROWS

GAUSSIAN ELIMINATION PIVOT METHOD

ELEMENTARY ROW OPERATIONS

1. SCALE: MULTIPLY A ROW BY A NON-ZERO CONSTANT
2. REPLACE: ADD A MULTIPLE OF A ROW INTO ANOTHER ROW
3. SWAP: SWITCH CONTENTS OF 2 ROWS

$$R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 4 & 3 & 1 \\ -1 & 2 & 8 \end{bmatrix}$$

PIVOT

SWAP SO THAT PIVOT IS ON THE TOP ROW

$$(-1)R_1 \begin{bmatrix} -1 & 2 & 8 \\ 4 & 3 & 1 \end{bmatrix}$$

MULTIPLY BY RECIPROCAL OF PIVOT'S CURRENT VALUE

SCALE SO THAT PIVOT EQUALS 1

$$\begin{bmatrix} 1 & -2 & -8 \\ 4 & 3 & 1 \end{bmatrix}$$

$$(-4)R_1 + R_2$$

MULTIPLY PIVOT'S ROW BY NEGATIVE OF ENTRY TO BE ELIMINATED

REPLACE EACH SUBSEQUENT ROW BY ADDING A MULTIPLE OF PIVOT'S ROW TO GET 0'S UNDER THE PIVOT

$$\left(\frac{1}{11}\right)R_2 \begin{bmatrix} 1 & -2 & -8 \\ 0 & 11 & 33 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & -2 & -8 \\ 0 & 1 & 3 \end{bmatrix}}_{\text{IN ROW ECHELON FORM}}$$

$x-2y=-8$

$y=3$

$x-2(3)=-8$

$x=-2$

NOT IN REDUCED ROW ECHELON FORM

GIVEN A MATRIX IN REF,
TO GET IT INTO RREF

$$(2)R_2 + R_1 \begin{bmatrix} 1 & -2 & -8 \\ 0 & 1 & 3 \end{bmatrix}$$

REF

START @ LOWEST PIVOT ROW
REPLACE EACH ROW ABOVE THE PIVOT
BY ADDING A MULTIPLE OF THE PIVOT ROW
TO GET 0 IN ALL OTHER ENTRIES OF THE PIVOT'S COLUMN

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix} \quad \begin{aligned} x &= -2 \\ y &= 3 \end{aligned}$$

RREF

Row Echelon Form (REF)

A matrix is in row echelon form if and only if
the first (leftmost) non-zero entry in each row is 1 (called the leading 1),
the leading 1 in each row (except row 1) is to the right of the leading 1 in the row above it,
and all rows which contain only 0 are below all rows which contain any non-zero entry.

A matrix in REF corresponds to a system of equations that needs only back-substitution to solve.

Are these matrices in REF ? If not, why not ?

NOT REF #1

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 0 & -1 & 5 & 6 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

NOT REF / NOT RIGHT OF L.E. IN R2

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 1 & 4 & -3 & -2 \\ 0 & 0 & 1 & 1 & 3 \end{bmatrix}$$

REF

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

REF

$$\begin{bmatrix} 1 & 3 & 0 & -2 & 4 \\ 0 & 1 & 7 & 4 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Row Echelon Form (RREF)

A matrix is in reduced row echelon form if and only if
it is in row echelon form,
and all columns which contain a leading 1 contain only 0 in all other entries.

A matrix in RREF corresponds to a system of equations that needs the least amount of algebra to solve.

Are these matrices in RREF ? If not, why not ?

NOT RREF

$$\begin{bmatrix} 1 & 0 & -1 & -2 & 4 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

RREF

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 4 \\ 0 & 1 & 8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF

ALL IN REF

Gaussian Elimination Pivot Method

Step 1: Find the first (leftmost) column which contains a non-zero entry
Step 2: Choose a pivot in that column (to be used to replace all lower entries in that column with 0)
Step 3: SWAP to move the pivot's row to the top
Step 4: SCALE to turn the pivot into 1
Step 5: REPLACE each row below the pivot's row
by adding the multiple of the pivot's row which gives a 0 under the pivot
Step 6: Cover up the pivot's row & repeat the entire process (stop when matrix is in row echelon form)

Gauss-Jordan Elimination (after matrix is in row echelon form)

Step 7: Find the last (rightmost) column which contains a pivot (leading 1)
Step 8: REPLACE each row above the pivot's row
by adding the multiple of the pivot's row which gives a 0 above the pivot
Step 9: Cover up the pivot's row & repeat the entire process (stop when matrix is in reduced row echelon form)

The following examples should not require fractions if solved using the processes above.

Example 1:

$3x + 2y - z = -1$
$5x + y - 3z = -2$
$2x + 4y + 2z = 2$

Example 2:

$4x + 6y - 3z = -5$
$3x + 4y + z = 11$
$-x - 2y + z = 1$

Example 3:

$3x + 4y - 11z = -17$
$2x + y - 4z = 5$
$-x - 2y + 5z = -9$

Example 4:

$3x + 5y - 9z = 14$

Example 5: